

Assessment: Assignment 4
Due: 21 October 2022
Marks: 100
Value: 10%

1 (worth 100 marks)

Introduction

The concentrated force P at the right end of the switch in Assignment 3 is now replaced with the following distributed force q :

$$q(x) = \begin{cases} 0, & 0 \leq x < L/2, \\ \frac{2(q_R - q_L)}{L}x + 2q_L - q_R, & L/2 \leq x \leq L, \end{cases} \quad (1)$$

where q_L and q_R are the left and right values of the distributed force q on the interval between $x = L/2$ and $x = L$ (Figure 1).

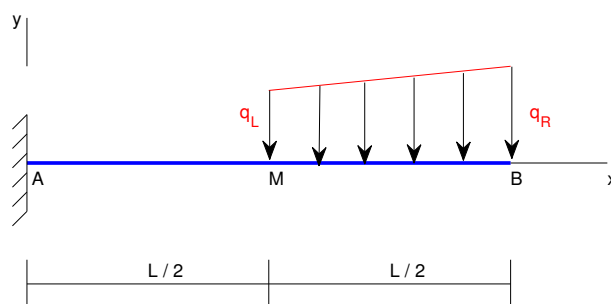


Figure 1: Cantilever beam: distributed force.

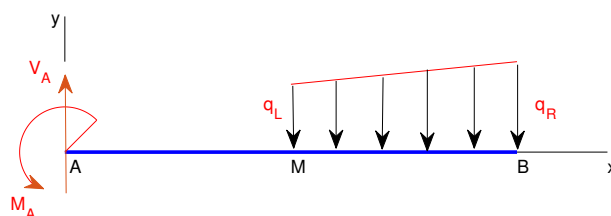


Figure 2: Cantilever beam: external forces.

To keep the switch in equilibrium, the fixed support at A provides the vertical reaction force

$$V_A = -\frac{L}{4}q_L - \frac{L}{4}q_R, \quad (2)$$

and the couple moment

$$M_A = -\frac{L^2}{6}q_L - \frac{5L^2}{24}q_R, \quad (3)$$

which are displayed in Figure 2. Under these external forces, there exist forces inside the switch, namely the shear force, denoted by V , and the bending moment, denoted by M . The mathematical relationships between $q(x)$, $V(x)$ and $M(x)$ are

$$\frac{dV}{dx} = q(x), \quad 0 \leq x \leq L, \quad (4)$$

$$\frac{dM}{dx} = V(x), \quad 0 \leq x \leq L. \quad (5)$$

One can establish the following ordinary differential equation to mathematically model the internal forces M and V :

$$\frac{d^2M}{dx^2} = q(x), \quad (6)$$

which is subject to the initial values

$$M(0) = -M_A, \quad (7)$$

$$V(0) = V_A. \quad (8)$$

Your tasks are to numerically solve the initial-value problem, defined by (6)-(8), and then compare the obtained results with the analytical solution.

For your assignment, the following values, which are not to be shared with anyone else, are to be used:

$$L = 5 \text{ cm}, \quad (9)$$

$$q_L = -100 - [6.4156 \times 5] \text{ N/m}, \quad (10)$$

$$q_R = -300 + [2.1766 \times 5] \text{ N/m}. \quad (11)$$

Requirements

For this assessment item, you must perform hand calculations:

1. Derive the analytical solutions V and M of the initial-value problem.
2. Employ the Euler method to solve the initial-value problem by using a set of 4 equal intervals to represent the continuous domain $[0, L]$. Report the solutions V and M rounded to 3 significant figures.
3. Calculate the absolute errors of the Euler method, which are defined as

$$\epsilon_V = |V_{approx} - V_{exact}|, \quad (12)$$

$$\epsilon_M = |M_{approx} - M_{exact}|, \quad (13)$$

for each grid point $x_i > 0$ in Requirement 2. In (12), V_{approx} and V_{exact} are the approximate and exact values of the shear force, and in (13), M_{approx} and M_{exact} are the approximate and exact values of the bending moment. Report the answers rounded to 3 significant figures.

You must also produce MATLAB code which:

4. Repeats the hand calculations in Requirements 2 and 3. Verify the answers by using the reported results from Requirement 2.
5. Employs the Euler method to solve the initial-value problem for the following three cases: 4, 10 and 100 equal intervals are used to represent the continuous domain $[0, L]$.
6. Plots the three approximate distributions of V from Requirement 5 and the corresponding analytical solution on the same graph. Discuss the results.
7. Plots the three approximate distributions of M from Requirement 5 and the corresponding analytical solution on the same graph. Discuss the results.

8. To find the order of accuracy of the Euler method, calculates the following relative error for each case in Requirement 5

$$\epsilon_{N_i} = \frac{\sqrt{\sum_{i=1}^{N_i} [(V_i - V_i^e)^2 + (M_i - M_i^e)^2]}}{\sqrt{\sum_{i=1}^{N_i} [(V_i^e)^2 + (M_i^e)^2]}}, \quad (14)$$

where superscript e denotes the exact solution, $V_i = V(x_i)$ and $M_i = M(x_i)$ (approximate values), $V_i^e = V^e(x_i)$ and $M_i^e = M^e(x_i)$ (exact values), $i = (1, 2, \dots, N_i)$, and N_i is the number of grid points. It is noted that $N_i = 5$ for the first data set, 11 for the second data set and 101 for the third data set. Construct a power function to model the behaviour of ϵ_{N_i}

$$\epsilon_{N_i} = bh_{N_i}^m, \quad (15)$$

where $h_{N_i} = L/(N_i - 1)$ (the grid size), and b and m are the two coefficients to be found. One can utilise the 3 data points: $(h_1 = L/(N_1 - 1), \epsilon_{N_1})$, $(h_2 = L/(N_2 - 1), \epsilon_{N_2})$ and $(h_3 = L/(N_3 - 1), \epsilon_{N_3})$ to construct the model function. Plot function $\epsilon_{N_i}(h_{N_i})$ on log-log axes and report value of m (m : the order of accuracy of the Euler method).

9. Solves the initial-value problem by using Matlab solver `<ode45>`. Graphically compare the distributions of M and V with the corresponding analytical ones.
10. Solves the initial-value problem by `<simulink>`. Plot the approximate solutions $V(x)$ and $M(x)$.
11. Discusses the case where the continuous domain $[0, L]$ is also divided into a set of equal intervals, but the number of intervals (i.e. $N_i - 1$) is an odd number.
12. Has appropriate comments throughout your Matlab program

You must submit a short video (10–30 seconds) where you discuss:

1. the part of the code of which you are most proud; and
2. the part of the code which you found most difficult to get working correctly.

Your video must show the relevant part(s) of the code while you are discussing them. Your video will be marked based on whether one is submitted or not: full marks for this item if a video is submitted; zero marks if a video is not submitted. Your video should not be in full HD format (i.e. your video should be no more than 1280 pixels across), so that the file size does not become too large.

Assessment Criteria

Your code will be assessed using the following scheme. Note that you are marked based on how well you perform for each category, so the correct answer determined in a basic way will receive half marks and the correct answer determined using an excellent method/code will receive full marks.

Quality of hand calculations	20 marks
Quality of Requirement 4	5 marks
Quality of Requirements 5-7	20 marks
Quality of Requirement 8	15 marks
Quality of Requirement 9	10 marks
Quality of Requirement 10	10 marks
Quality of Requirement 11	10 marks
Quality of header(s), comments and reporting	5 marks
Submission of video	5 marks

Submission

Submit your Matlab program by the due date to the StudyDesk. The Matlab program includes a single m file that covers all relevant requirements and its associated function files (if existed), and your simulink file. Submit your hand calculations as a pdf file, and your video as a standard video file. If your video is too big, convert it to a lower resolution using Handbrake (<https://handbrake.fr/>) or something similar. The maximum submission size is 5 MB.

- You do not need to rename your files when uploading: the system automatically segregates different students' submissions.
- If you can see that the files have uploaded, then you have successfully submitted your assignment. There is no need to click a "send for marking" button, but you will have to click a button confirming that the submission is your own work.
- You **MUST** upload all of your code, your pdf file and your video file in a *.zip file. You do not need to install any special software to be able to do this. In Windows: right-mouse-click on the file(s)/folder(s) that you want to include, then choose "Send to" → "Compressed (zipped) folder". The following are the only file types that can be submitted:
 - *.zip
 - *.pdf
 - *.doc
 - *.docx
 - standard video formats

The system will block any attempt by you to upload a file which doesn't match any of those file extensions.

- After the submission deadline, if you forgot to submit a file, do not upload it after the due date: the submission time is based on when the last file was uploaded. You should email the examiner in this circumstance (with any file attached).